

MOST Brolect 251200 MA0 6739 USL Problem No. 1-410-00-00 U. S. NAVY UNDERWATER SOUND LABORATORY FORT TRUMBULL, NEW LONDON, CONNECTICUT SPECTRA AND WAVEFORMS OF BOTTOM REFLECTED PULSES. Technical Edward S. Eby 1-160-66 USL Technical Memorandum No. 914-160-66 10 June 1966 ABSTRACT<sup>2</sup> A mathematical model considering reflection as a reradiation phenomenon has been constructed. The model allows computation of the

spectrum and waveform of a reflected pulse in terms of the incident plane wave pulse or its spectrum, the reradiation characteristics of the reflector, the incident direction and the reradiation direction. Bottom reflection of sonic pulses in an isovelocity medium has been considered in terms of this model and temporal and spectral distortions have been calculated. The theory of Cron and Nuttall (J. Acoust. Soc. Am., 37, 486-492(1965)) is shown to be a special case of the present theory.

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1. This Memorandum consists of the abstract, test and slides of a paper read to the Seventy-first Meeting of the Acoustical Society of America in Boston, Massachusetts, on 2 June 1966. 2.

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### SPECTRA AND WAVEFORMS OF BOTTOM REFLECTED PULSES

E. S. EBY

#### INTRODUCTION

IN THIS PAPER,

SOME RESULTS DERVIED FROM A MATHEMATICAL MODEL

CONSIDERING BOTTOM REFLECTION AS A RERADIATION PHENOMENON

ARE PRESENTED.

THE PULSES BEING REFLECTED

ARE TRANSMITTED FROM A DIRECTIONAL SOURCE
AND REPADLATED FROM THE BOTTOM.

THEORETICAL RESULTS WILL BE DISCUSSED FIRST.

IT WILL BE SHOWN

THAT FOR A UNIFORMLY RERADIATING BOTTOM,
ONLY SPECULAR REFLECTION

AND SNELL'S LAW REFRACTION OCCUR.

HOWEVER, FOR NONUNIFORM RERADIATION,

ENERGY IS RERADIATED IN OTHER DIRECTIONS AS WELL.

THIS THEORY IS THEN APPLIED

TO A TWO SHIP SITUATION

WHERE THE SOURCE SHIP HAS A DIRECTIONAL TRANSMITTER.

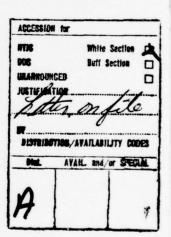
IT WILL BE SHOWN

THAT MOTION OF THE SOURCE SHIP

CAUSES PERTERBATIONS OF THE RECEIVED SPECTRUM

AS WELL AS PULSE SHAPE DEGRADATION

AND TIME SMEARING.



IN THIS SLIDE, THE BASIC SITUATION IS SHOWN.

THE VECTORS / , M, N REPRESENT THE NORMALS

TO THE INCIDENT PLANE WAVES,

THE BOTTOM,

AND THE RERADIATED PLANE WAVES, RESPECTIVELY.

THE PLANE OF THE BOTTOM

IS ASSUMED TO BE THE PLANE  $x_3 = 0$ FOR CONVENIENCE IN CALCULATION.

IT HAS ALSO BEEN ASSUMED

THAT THE PULSE TRANSMITTED BY A DIRECTIONAL SOURCE

LOCATED AT THE UPPER LEFT,

OFF THE SLIDE,

CAN BE SEPARATED INTO TWO PARTS:

THE TRANSMITTED WAVEFORM,

AND THE PROJECTION OF THE BEAM PATTERN OF THE SOURCE ONTO THE BOTTOM WHICH IS CALLED THE REPADIATION FUNCTION.

THE RERADIATION FUNCTION

IS ASSUMED TO BE NORMALIZED,

AGAIN FOR CALCULATIONAL CONVENIENCE.

SINCE THE BOTTOM IS THE PLANE x3 = 0

THE RERADIATION FUNCTION IS ACTUALLY A FUNCTION

ONLY OF  $x_1$  AND  $x_2$ .

THE SOUND SPEED IN THE UPPER MEDIUM IS c.

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THE PLANE WAVES RERADIATED IN THE DIRECTION N
TRAVEL WITH VELOCITY v.

IF N IS DIRECTED INTO THE UPPER MEDIUM
THEN v IS EQUAL TO c,

BUT IF DIRECTED INTO THE LOWER MEDIUM,
v WILL DIFFER FROM c.

AND THE RERADIATION FUNCTION IS DENOTED BY w(X),

THEN THE AMPLITUDE OF A PARTICULAR RERADIATED PLANE WAVE

IS OBTAINED BY SUMMING THE CONTRIBUTIONS

FROM THE PULSE INCIDENT ON THE BOTTOM

TO A PLANE TRAVELING WITH VELOCITY v IN DIRECTION N

WEIGHTED BY THE RERADIATION FUNCTION.

THIS AMPLITUDE IS EXPRESSED BY g(t,p) AS SHOWN.

THE VECTOR P IS INTRODUCED TO SIMPLIFY THE NOTATION

AND REPRESENTS THE COMBINED EFFECTS OF THE INCIDENT DIRECTION,

THE RERADIATED DIRECTION AND THE CORRESPONDING VELOCITIES.

THEREFORE WE SEE THAT THE RERADIATED WAVEFORM CAN BE EXPRESSED

WE LET  $f(\omega)$  AND  $G(\omega,P)$  BE THE STANDARD FOURIER TRANSFORMS OF THE INCIENT WAVEFORM,

AS A CONVOLUTION OF THE INCIDENT PULSE WITH THE RERADIATION FUNCTION.

AND THE RERADIATED WAVEFORM.

WE DEFINE THE TRANSFER FUNCTION OF THE BOTTOM

TO BE THE "FOURIER" TRANSFORM, CAP  $w(\infty, P)$ ,

OF THE RERADIATION FUNCTION, w(x), AS SHOWN HERE.

WE ALSO DEFINE THE IMPULSE RESPONSE OF THE BOTTOM w(t, P)

TO BE THE TIME FUNCTION OBTAINED

FROM A STANDARD FOURIER INVERSION

APPLIED TO THE FREQUENCY FUNCTION CAP W( , P).

NOTE THAT THE QUANTITIES ON THE LEFT HAND SIDES OF THESE EQUATIONS

ARE DEFINED BY THE EXPRESSIONS ON THE RIGHT-HAND SIDES.

# USING THESE DEFINITIONS

# OF TRANSFER FUNCTION AND IMPULSE RESPONSE

IT CAN BE SHOWN

THAT THE USUAL THEOREMS FROM COMMUNICATION THEORY SHOWN HERE
HOLD IN THIS MODEL.

THESE THEOREMS SHOW

WE CAN STUDY THE EFFECT OF A DIRECTIONAL SOURCE
BY STUDYING THE PROPERTIES
OF THE TRANSFER FUNCTION AND IMPULSE RESPONSE.

IF, IN ADDITION TO THE BEAM PATTERN EFFECTS WE HAVE DISCUSSED,

THE BOTTOM INTRODUCES

THE PHASE SHIFT AND ATTENUATION TERM

DISCUSSED IN THE PRECEEDING PAPER,

THE TRANSFER FUNCTION MUST BE MULTIPLIED

BY THE TERM e je sgnw -b(w)

AND THE IMPULSE RESPONSE

MUST BE CONVOLVED WITH THE CORRESPONDING TIME FUNCTION.

IF THE PROJECTION OF THE BEAM PATTERN ONTO THE BOTTOM

IS THIS ELLIPTIC PARABOLOID,

THEN THE TRANSFER FUNCTION

IS 8 Jr (Rec)

THE PARAMETER k HERE

EXPRESSES THE EFFECT

OF THE BOTTOM AREA ILLUMINATED

AS WELL AS THE DIRECTIONS AND VELOCITIES.

THE LIMIT AS a AND a GO TO INFINITY,

IS THE TRANSFER FUNCTION

FOR A UNIFORMLY REPADLATING BOTTOM.

THE LIMIT IS UNITY

OF THE VECTOR P ARE ZERO

AND ZERO OTHERWISE.

IF WE NOW GO BACK

TO THE DEFINITION OF THE VECTOR P

AND SEE WHAT THE CONDITIONS  $p_1 = p_2 = 0$  IMPLY,

WE FIND THAT THESE CONDITIONS

ARE EQUIVALENT TO SPECULAR REFLECTION

OR SNELL'S LAW REFRACTION.

DEPENDING ONLY ON WHETHER THE RERADIATION DIRECTION

IS UPWARD OR DOWNWARD.

THIS IS THE CASE

FOR WHICH PHASE SHIFT

AND ATTENUATION EFFECTS OF THE BOTTOM

HAVE BEEN CONSIDERED BY CRON AND NUTTALL.

THE ELLIPTIC PARABOLOID SHOWN HERE

IS THE RERADIATION FUNCTION ASSUMED

FOR THE SITUATION SHOWN IN THE NEXT SLIDE.

THE SOURCE AND RECEIVING SHIPS ARE POSITIONED

SO THE TRANSMITTED PULSE WILL ARIVE BY SPECULAR REFLECTION

FOR A 30° DEPRESSION ANGLE.

THE DIRECTIONAL SOURCE

HAS A 6° HORIZONTAL BEAM WIDTH

AND A 10° VERTICAL BEAM WIDTH.

THE RECEIVING TRANSDUCER IS OMNIDIRECTIONAL
TO AVOID EFFECTS OF RECEIVING SHIP MOTION.

WE NOW WILL SHOW THE EFFECT

OF A GENTLE ROLL,

ONLY 50

ON A 200 MILLISECOND RECTANGULAR PULSE ENVELOPE.

A OO ROLL GIVES SPECULAR REFLECTION

AND IS SHOWN IN WHITE.

THE BEAM FOR 2-50 ROLL

IS SHOWN IN BLUE

AND FOR +50, IN RED.

THIS COLOR CODING IS USED

IN THE SLIDES WHICH FOLLOW.

THE ROLLING MOTION OF THE SOURCE IS ASSUMED TO BE SUFFICIENTLY SLOW

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THAT THESE THREE SITUATIONS

CAN BE CONSIDERED STATIC

AND DYNAMIC EFFECTS ARE IGNORED.

THE UPPER CURVES ARE THE TRANSFER FUNCTIONS FOR THE ROLL ANGLES OF 0 $^{\circ}$  AND  $\pm 5^{\circ}$  AND ARE SEEN TO CHANGE RATHER DRASTICALLY.

HOWEVER, FROM THE BOTTOM FIGURE

WE SEE THAT THE SPECTRA OF THE RECEIVED WAVEFORMS

ARE NOT APPRECIABLY CHANGED

EXCEPT THAT THE HIGHER FREQUENCY COMPONENTS

ARE EFFECTIVELY MISSING

FROM THE TRANSMITTED SPECTRUM.

OF COURSE, THE ZERO ROLL CURVE

IS EXACTLY THE TRANSMITTED SPECTRUM.

THIS FIGURE SHOWS THE CHANGE IN WAVESHAPE

OF THE RECEIVED SIGNALS

AS A FUNCTION OF ROLL ANGLE.

THE PULSE SHAPES FOR NON-SPECULAR RERADIATION

ARE DEGRADED AND SHOW A LARGE TIME SMEAR.

THE PULSES RECEIVED FOR -5 AND +5° ROLLS

HAVE DURATIONS OF 341

AND 574 MILLISECONDS

COMPARED TO THE TRANSMITTED PULSE DURATION

OF 200 MILLISECONDS.

THIS CORRESPONDS

TO AN UNATTENUATED -ZERO DEGREE PHASE SHIFT

IN TERMS OF THE PRECEEDING PAPER.

THE LAST SLIDE

SHOWS THE ADDITIONAL PULSE SHAPE DEGRADATION

FOR AN UNATTENUATED - 60° PHASE SHIFT

ASSUMING THE RESULTS OF CRON AND NUTTALL.

IN SUMMARY, WE SEE THAT

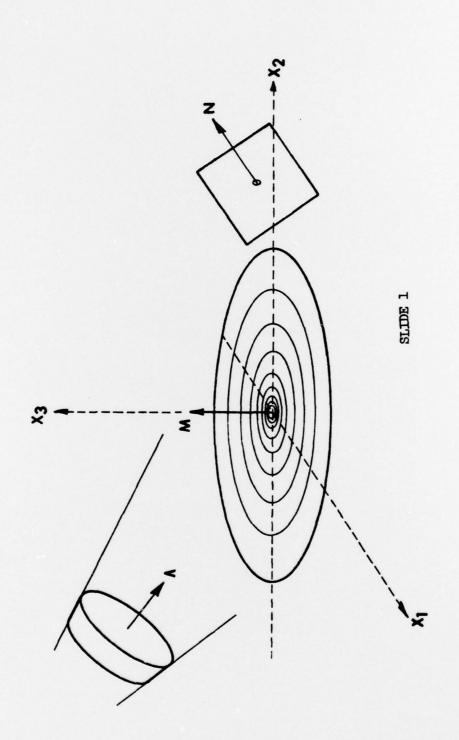
THIS RERADIATION POINT OF VIEW

ALLOWS THE COMPUTATION OF SPECTRA AND WAVEFORMS

RECEIVED FROM A DIRECTIONAL SOURCE

WHICH SHOWS THE TIME SMEAR AND PULSE SHAPE DEGRADATION

CHARACTERIZING PULSES RECEIVED IN EXPERIMENTAL SITUATIONS.



RERADIATED WAVEFORM:

$$g(t,P) = \int_{B} f\left(t - \frac{X \cdot P}{c}\right) w(X) dx$$

TRANSFER FUNCTION:

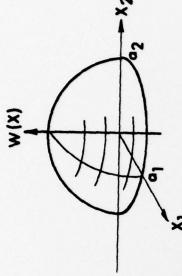
$$W(\omega, P) = \int_{B} w(X) e^{-j\frac{\omega}{C}X \cdot P} dx$$

IMPULSE RESPONSE:

$$w(t,P) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W(\omega,P) e^{+j\omega t} d\omega$$
THEOREMS:

I. 
$$G(\omega, P) = F(\omega) W(\omega, P)$$
  
 $\Pi$ .  $g(t, P) = \int_{-\infty}^{\infty} f(\tau) W(t-\tau, P) d\tau$ 

SLIDE 2



TRANSFER FUNCTION: XI

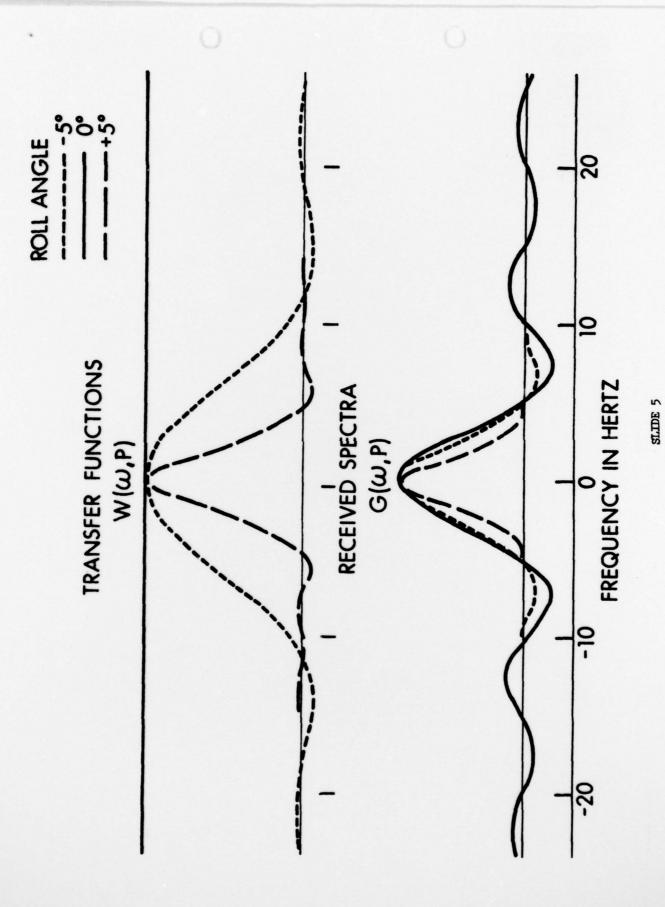
$$W(\omega,P) = 8 \frac{J_2(k\omega)}{(k\omega)^2}$$

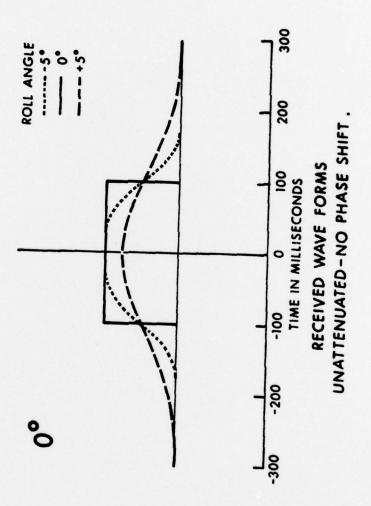
UNIFORMLY RERADIATING BOTTOM:

$$\lim_{\alpha_1 \to \infty} W(\omega, P) = \begin{cases} 1, P_1 = P_2 = 0 \\ 0, \text{ OTHERWISE} \end{cases}$$

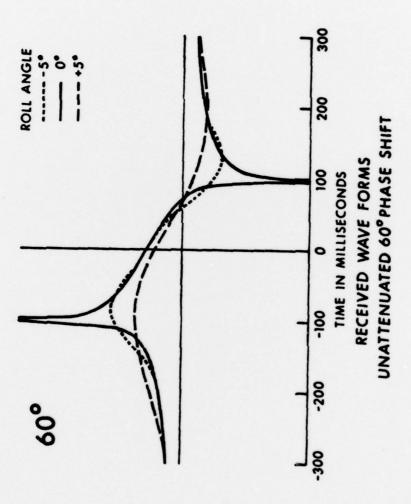
SLIDE 3

SLIDE 4





SLIDE 6



SLIDE 7